Slothrop: Knuth-Bendix Completion with Modern Termination Checking

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A Good Order is Hard to Find

- Input to completion: set of identities and a **compatible reduction order**.
- Compatible orders hard to find.
- Compatible orders hard to specify.
  - Tools implement only a few classes of orders.

Main Idea

- Use a **termination checker** instead.
  - User doesn't explicitly provide a compatible reduction order.
  - Ensures termination of intermediate rewrite systems.
- Goal: help **complete more theories** in practice.

Completion Refresher

- Completion as an inference system:

ORIENT:	$\dfrac{(E \cup \{s \dot{\approx} t\}, R)}{(E, R \cup \{s \to t\})}$	if $s > t$
DEDUCE:	$\dfrac{(E, R)}{(E \cup \{s \approx t\}, R)}$	if $s \leftarrow_R u \rightarrow_R t$
DELETE:	$\dfrac{(E \cup \{s \approx s\}, R)}{(E, R)}$	
SIMPLIFY:	$\dfrac{(E \cup \{s \dot{\approx} t\}, R)}{(E \cup \{u \dot{\approx} t\}, R)}$	if $s \rightarrow_R u$
COMPOSE:	$\dfrac{(E, R \cup \{s \to t\})}{(E, R \cup \{s \to u\})}$	if $t \rightarrow_R u$
COLLAPSE:	$\dfrac{(E, R \cup \{s \to t\})}{(E \cup \{v \approx t\}, R)}$	if $s \xrightarrow{\sqsupset}_R v$

Modified Orient Rule

- Tentative change to **orient** rule precondition:

$$\textsc{orient:}\quad \frac{(E \cup \{s \doteq t\}, R)}{(E, R \cup \{s \to t\})} \qquad \text{if } s > t$$

← Original

Modified

↓

$$\textsc{orient:}\quad \frac{(E \cup \{s \doteq t\}, R)}{(E, R \cup \{s \to t\})} \qquad \text{if } R \cup \{s \to t\} \text{ terminates}$$

Multiple Orders

- Equivalent formulations?
  - Compatible order $\Rightarrow$ termination.
  - Termination $\Rightarrow$ compatible order exists.
- No: could be a **different order** for each application of orient rule.
- Completion with multiple orders **not correct** – counterexamples by Sattler-Klein.

Refining Orders

- Obs: if intermediate rewriting systems terminate and form an **increasing chain**, an increasing chain of compatible orders exists.
- The **one** final order of the chain is compatible with **all intermediate systems**.

Constraint Systems

- Removing redundant rules important for performance though.
- Fix: use a **constraint** rewriting system *C*.

1. Intermediate constraint systems form increasing chain.
2. *C* terminates ⇒ *R* terminates.

- *C* is like *R*, but rules are only added.

Modified Completion

- Completion with termination checking:

$$\textsc{orient:}\quad \frac{(E \cup \{s \mathrel{\dot{\approx}} t\}, R, C)}{(E, R \cup \{s \to t\}, C \cup \{s \to t\})} \quad \text{if } C \cup \{s \to t\} \text{ terminates}$$

$$\textsc{deduce:}\quad \frac{(E, R, C)}{(E \cup \{s \approx t\}, R, C)} \quad \text{if } s \leftarrow_R u \to_R t$$

$$\textsc{delete:}\quad \frac{(E \cup \{s \approx s\}, R, C)}{(E, R, C)}$$

$$\textsc{simplify:}\quad \frac{(E \cup \{s \mathrel{\dot{\approx}} t\}, R, C)}{(E \cup \{u \mathrel{\dot{\approx}} t\}, R, C)} \quad \text{if } s \to_R u$$

$$\textsc{compose:}\quad \frac{(E, R \cup \{s \to t\}, C)}{(E, R \cup \{s \to u\}, C)} \quad \text{if } t \to_R u$$

$$\textsc{collapse:}\quad \frac{(E, R \cup \{s \to t\}, C)}{(E \cup \{v \approx t\}, R, C)} \quad \text{if } s \stackrel{\sqsupset}{\to}_R v$$

Completion Search

- What if an identity can be oriented in either direction?
- Just try both ways – **search** for a convergent completion.
- Succeeds if search is fair (e.g., breadth-first).

Implementation

- Completion with termination checking implemented: **Slothrop**.
- ~7k line Ocaml program, integrates with **AProVE**.
- Applications of orient rule preceded with calls to AProVE to verify termination of constraint system.

Slothrop's Completions

- Slothrop automatically finds completions for small theories (groups, etc.) and some larger theories (> 20 eqs).
- **First automatic completion** of theory of two commuting group endomorphisms (CGE_2).
- Time: ~6s groups, ~1hr CGE_2.

Conclusion

- Details in tech report WUCSE-2006-45.
- Slothrop available online at cl.cse.wustl.edu.
- Thanks to AProVE team for help with integration.